§25. Dynamical Equation of Turbulence in Heliotron/torsatron

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Recently, theory of the plasma turbulence has progressed. By use of the method of dressed test mode, a dynamical equation is derived for the plasmas in Heliotron/torsatron configurations. We study a plasma with magnetic hill in the cylindrical model. The reduced set of equation is employed. Lagrangean nonlinearlities are renormalized, and dynamical equation for the dressed test mode has been derived in a form as

$$\frac{d}{d\hat{k}}\frac{\hat{k}_\perp^2}{\hat{\gamma}+\hat{\lambda}\hat{k}_\perp^4}\frac{d}{d\hat{k}}L_\chi p+\frac{G_0}{s^2}p - \frac{1}{\hat{k}_\theta^2 s^2}L_\mu\frac{d}{d\hat{k}}L_\chi p=0 \tag{1}$$

where $L_\mu \equiv \hat{\gamma}+\hat{\mu}\hat{k}_\perp^2+\hat{k}_\theta\omega_{E1}d/d\hat{k}$, $L_\chi \equiv \hat{\gamma}+\hat{\mu}\hat{k}_\perp^2+\hat{k}_\theta\omega_{E1}d/d\hat{k}$ [1]. In writing Eq.(1), the symbol hat denotes the normalized quantity. We use the normalization $r/a \rightarrow \hat{r}$, $t/\tau_{Ap} \rightarrow \hat{t}$, $\mu\tau_{Ap}/a^2 \rightarrow \hat{\mu}$, $\chi\tau_{Ap}/a^2 \rightarrow \hat{\chi}$, $\lambda\tau_{Ap}/\mu_0 a^4 \rightarrow \hat{\lambda}$, $\tau_{Ap}=a\sqrt{\mu_0 m_i n_i}/B_p$, $\gamma\tau_{Ap} \rightarrow \hat{\gamma}$, $\omega_{E1}=\tau_{Ap}E_r'/B$, γ is the growth rate, $G_0 = \Omega'\beta'/2\varepsilon^2$ indicates the pressure gradient, the shear parameter s is defined by the relation $k_{\parallel} = kqs(r-r_s)/qR$, $q(r_s) = m/n$, and others are standard. The transport coefficient and the fluctuation amplitude have the relation as $\mu = (\tilde{\phi}/2B)^2 k_\perp^2 (\gamma+\mu k_\perp^2)^{-1}$ and $\hat{\chi} \simeq \hat{\mu}$.

The temporal evolution in the vicinity of the stationary state is examined. Equation (1) is expanded with respect to $\hat{\gamma}$. Keeping the first order correction with respect to $\hat{\gamma}$, we have the equation like $\hat{\gamma}(L_{t0}+\omega_{E1}L_{t1})\tilde{p}=(L_0+\omega_{E1}L_1)\tilde{p}$, where explicit form of operators L_0, L_1, L_{t0} and L_{t1} are given in [2]. The growth rate $\hat{\gamma}$ as well as the effect of the radial electric field inhomogeneity is calculated by expanding in a series of Weber functions. After straightforward calculations, we have

$$\hat{\gamma}=\frac{0.45\sqrt{G_0}}{(1+0.02G_0^{-1}\omega_{E1}^2)}\left(1-\frac{\hat{\chi}}{\hat{\chi}_{H0}}\right) \tag{2}$$

where $\hat{\chi}_{H0}$ denotes the transport coefficient in a steady state. The stationary solution has been obtained in [1] and the transport coefficient, including the effects of the radial electric field inhomogeneity, is given as $\hat{\chi}_{H0} \equiv G_0^{3/2}c^2a^{-2}\omega_p^{-2}/(1+h_1\omega_{E1}^2)$.

In the small amplitude limit, we have

$$\hat{\gamma}\approx\frac{0.58\,G_0^{1/2}}{1+0.35G_0^{-1}\omega_{E1}^2}\left(\frac{\hat{\chi}}{\hat{\chi}_{H0}}\right)^{1/5} \tag{3}$$

Combining these results, a connection formula of the growth rate is given as

$$\hat{\gamma}\simeq\frac{0.5\,G_0^{1/2}}{1+0.35G_0^{-1}\omega_{E1}^2}\left(\frac{\hat{\chi}}{\hat{\chi}_{H0}}\right)^{1/5}\left(1-\frac{\hat{\chi}}{\hat{\chi}_{H0}}\right) \tag{4}$$

This formula provides a basis to study the dynamic evolution of the strong turbulence in the plasma.

1) K. Itoh, et al., Plasma Phys. Contr. Fusion 36 (1994) 123.
2) K. Itoh, et al., NIFS Report NIFS-360 (May, 1995).